THE SCIENCE ACADEMY

J1 MATH E-TRIAL

A LEVEL H2 MATHEMATICS

THE SCIENCE ACADEMY

AP/GP

A marathon runner and a sprinter are running around a circular track of length 400 m. The sprinter runs his first 200 m in 24 seconds, and the time he takes for each subsequent lap is 1 second more than the time taken for the previous lap. The marathon runner completes his first 400 m lap in 54 seconds, and the time he takes for each subsequent lap is 3% more than the time taken for the previous lap.

(i) Find the time taken by the marathon runner for his third lap. [2]

(ii) Find the time the marathon runner takes to complete n laps, giving your answer in terms of n. [2]

(iii) Find the time the sprinter takes to complete n laps, giving your answer in terms of n. [2]

AP/GP

The two runners decide to do a 10 km race with the same starting point.

(iv) Determine which runner is the first to complete the race. [3]

(v) Find out which lap the marathon runner is on when he overtakes the sprinter. [3]

SEQUENCES AND SERIES

It is given that $\mathrm{f}(r)=2r^3-3r^2+r-4$.

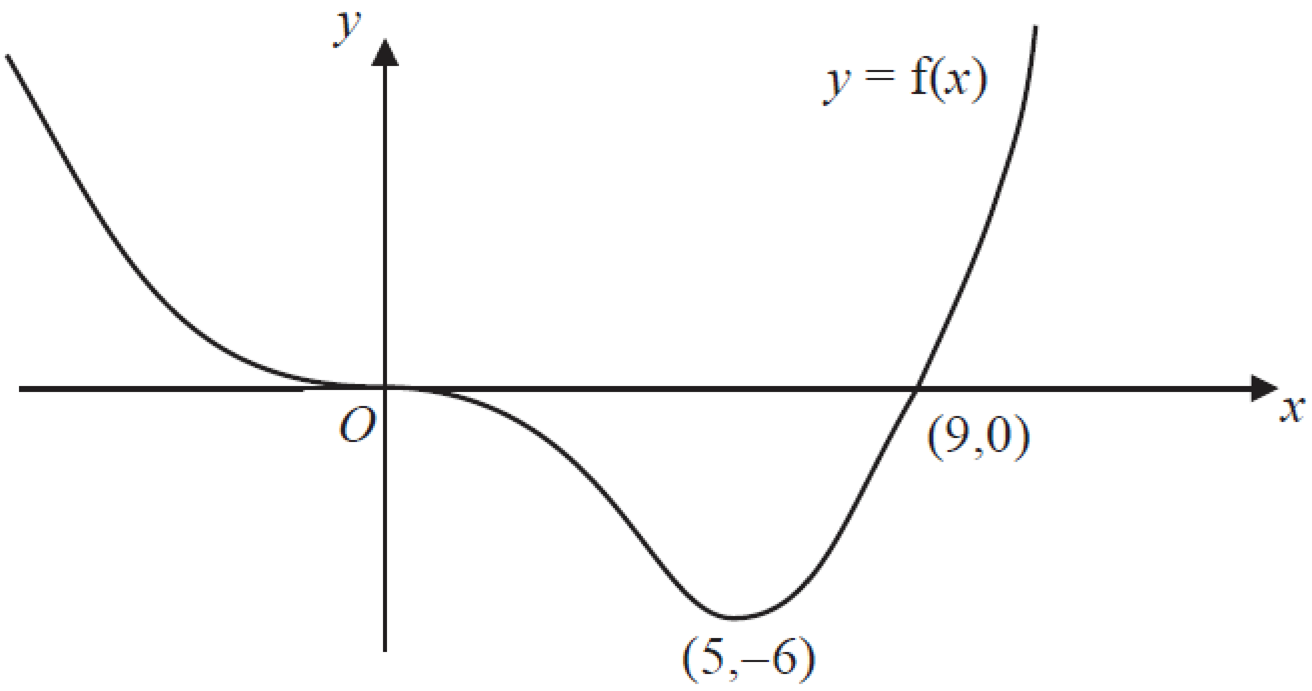
(i) Show that $\mathrm{f}(r+1)-\mathrm{f}(r)=kr^2$, where k is a constant to be determined. [2]

(ii) Deduce a formula for $\sum_{r=1}^{n} r^2$, fully factorising your answer. [3]

(iii) Hence show that $\sum_{r=4}^{n-3}(r-1)^2=\frac{1}{6}\mathrm{g}(n)-5$, where $\mathrm{g}(n)$ is to be determined. [3]

GRAPH SKETCHING

The diagram below shows the graph of $y = \mathrm{f}(x)$. It cuts the x-axis at (9,0) and has stationary points at the origin O and (5,–6).



GRAPH SKETCHING

On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}'(x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [2]

stating clearly the equations of any asymptotes, and the coordinates of any turning points and axial intercepts.

FUNCTIONS

The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{3x+4}{2x-3}, \quad x \in \mathbb{R}, x \neq \frac{3}{2},$$

$$\mathrm{g}: x \mapsto \mathrm{e}^{x}, \quad x \in \mathbb{R}.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$, stating the equations of the asymptotes and the coordinates of the points of intersection with the axes. Hence, show that f^{-1} exists.

[4]

(ii) Find $\mathrm{f}^{-1}(x)$. Hence or otherwise, find $\mathrm{f}^{3}(x)$. [4]

(iii) Find the range of gf. (You do not need to show that gf exists.) [2]

VECTORS

The planes p and q have equations $\mathbf{r}\cdot\begin{pmatrix}4\\2\\7\end{pmatrix}=-13$ and $\mathbf{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}h-1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}h+1\\-3\\2\end{pmatrix}$ respectively, where h is a constant and λ and μ are parameters.

(i) Find $\begin{pmatrix}h-1\\1\\2\end{pmatrix}\times\begin{pmatrix}h+1\\-3\\2\end{pmatrix}$ in terms of h. [2]

(ii) Find the value of h such that p and q are perpendicular. [2]

VECTORS

(iii) Given instead that p and q are parallel, find the perpendicular distance between p and q. [3]

(iv) In the case where $h = 0$, p and q intersect in a line l. The line l cuts the xz-plane at the point A and the yz-plane at the point B.

Find the position vectors of the points A and B. [4]

Hence find an equation that describes the set of all points which are equidistant from the points A and B. [3]